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nomial factors in x, y, z, with respect to the three coordinate planes through the centre of forces in each, be r_1, v_1 and u_1 for the first and r_2, v_2 and u_2 for the second, and let R, V and U be the radii for their product. Also let the radii for the two factors and their product, with respect to the centre of forces in each, be ρ_1, ρ_2 and ρ. The square of the distance of any coefficient from the centre of forces, is equal to the sum of the squares of its distances from three coordinate planes through that centre. We have then, by reasoning similar to that by which (44) was obtained,

$$\rho_1^2 = r_1^2+v_1^2+u_1^2, \qquad \rho_2^2 = r_2^2+v_2^2+u_2^2, \qquad \rho^2 = R^2+V^2+U^2. \tag{49}$$

But we have already found

$$R^2 = r_1^2+r_2^2, \qquad V^2 = v_1^2+v_2^2, \qquad U^2 = u_1^2+u_2^2, \tag{50}$$

whence finally,

$$\rho^2 = \rho_1^2+\rho_2^2. \tag{51}$$

Thus, in the product of two or any number of polynomials in x, y and z, the square of what we call the radius of gyration with respect to the centre of forces, is equal to the sum of the squares of the radii for all the factors with respect to their centres of forces.

(To be concluded in No. 4.)

PARALLEL CHORDS IN AN ELLIPSE.

BY PROF. ASAPH HALL.

LET φ be the eccentric angle of a point on the ellipse, or its eccentric anomaly. The rectangular coordinates of this point are

$$x = a\cos\varphi : \qquad y = b\sin\varphi.$$

Let the equation of a right line cutting the ellipse be

$$y = \alpha . x + \beta.$$

Denote by φ_1 the eccentric angle where the right line cuts the ellipse, and by φ_{10} the similar angle for the opposite point in which this line meets the ellipse. For a second right line we use φ_2 and φ_{20}. Hence we have

$$b\cos\tfrac{1}{2}(\varphi_{10}+\varphi_1) = -\alpha . a\sin\tfrac{1}{2}(\varphi_{10}+\varphi_1). \quad \text{etc.}$$

If $\varDelta_1$ be the length of the chord from φ_1 to φ_{10},

$$\varDelta_1^2 = (x_{10}-x_1)^2 + (y_{10}-y_1)^2,$$

or

$$\varDelta_1^2 = 4a^2 . (1+\alpha^2) . \sin\tfrac{1}{2}(\varphi_{10}+\varphi_1)^2 . \sin\tfrac{1}{2}(\varphi_{10}-\varphi_1)^2.$$

Similarly

$$\varDelta_2^2 = 4a^2 . (1+\alpha^2) . \sin\tfrac{1}{2}(\varphi_{20}+\varphi_2)^2 . \sin\tfrac{1}{2}(\varphi_{20}-\varphi_2)^2,$$

$$\Delta_3^2 = 4a^2.(1+a^2).\sin\tfrac{1}{2}(\varphi_{3\,0}+\varphi_3)^2.\sin\tfrac{1}{2}(\varphi_{3\,0}-\varphi_3)^2,$$
etc.

If we substitute in the equation of the right line the coordinates for the points $\varphi_{1\,0}$, φ_1, $\varphi_{2\,0}$, φ_2, and eliminate α and β, we have for the equation of the chords,

$$\frac{\cos\frac{1}{2}(\varphi_{1\,0}+\varphi_1)}{a}.x+\frac{\sin\frac{1}{2}(\varphi_{1\,0}+\varphi_1)}{b}.y-\cos\tfrac{1}{2}(\varphi_{1\,0}-\varphi_1)=0.$$

$$\frac{\cos\frac{1}{2}(\varphi_{2\,0}+\varphi_2)}{a}.x+\frac{\sin\frac{1}{2}(\varphi_{2\,0}+\varphi_2)}{b}.y-\cos\tfrac{1}{2}(\varphi_{2\,0}-\varphi_2)=0.$$

The condition that these chords are parallel is

$$\varphi_{1\,0}+\varphi_1=\varphi_{2\,0}+\varphi_2.$$

The length of the perpendicular on the first chord is

$$p_1=\frac{-b\cos\frac{1}{2}(\varphi_{1\,0}-\varphi_1)}{\sqrt{(1+a^2)}.\sin\frac{1}{2}(\varphi_{1\,0}+\varphi_1)}.$$

If we consider four parallel chords and denote the six differences of the perpendiculars by (1.2), (1.3), (1.4), (2.3), (2.4), (3.4), we shall have

$$(1.2)=\frac{b}{\sqrt{(1+a^2)}.\sin\frac{1}{2}(\varphi_{1\,0}+\varphi_1)}.\left\{\cos\tfrac{1}{2}(\varphi_{1\,0}-\varphi_1)-\cos\tfrac{1}{2}(\varphi_{2\,0}-\varphi_2)\right\}$$
$$=A.(c-d),$$

and $(1.3)=A.(c-e)$: $(1.4)=A.(c-f)$: $(2.3)=A.(d-e)$:
$(2.4)=A.(d-f)$: $(3.4)=A.(e-f)$,

in which A is a factor constant for parallel chords. It is plain that the values of Δ^2 may be written

$$\Delta_1^2=B.(1-c^2):\ \Delta_2^2=B.(1-d^2):\ \text{etc.},$$

where B is also constant for parallel chords.

If now we form the expression

$$+(2.3)(2.4)(3.4).\Delta_1^2-(1.3)(1.4)(3.4).\Delta_2^2$$
$$+(1.2)(1.4)(2.4).\Delta_3^2-(1.2)(1.3)(2.3).\Delta_4^2,$$

we have, after substitution, and the omission of constant factors,

$$+(d-e)(d-f)(e-f)(1-c^2)-(c-e)(c-f)(e-f)(1-d^2)$$
$$+(c-d)(c-f)(d-f)(1-e^2)-(c-d)(c-e)(d-e)(1-f^2),$$

and we shall find that this symmetrical function reduces to zero. That is, if we form the product of the square of the chord by the mutual distances of the three other chords, the sum of the products of the first and third is equal to the sum of the products of the second and fourth.

This theorem was announced by Savary (Conn. des Tems, 1830, p. 65), in a Memoir on the orbits of double stars. It will be seen that this property holds when the intersecting line is inclined to the chords.